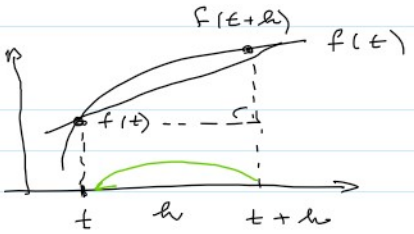
Konzultace k 11. přednášce

Diskretizace



$$\Delta_1 f = f[n+1] - f[n]$$

$$\Delta_2 f = \Delta_1(\Delta_1 f) =$$

$$= \Delta_1(f[n+1] - f[n]) =$$

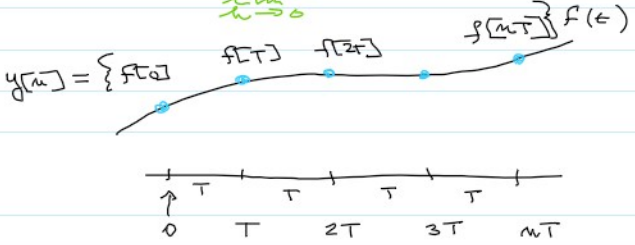
$$= f[n+2] - f[n+1] - (f[n+1] - f[n]) =$$

$$= f[n+2] - 2f[n+1] + f[n]$$

$$f'(t) \approx \frac{f[n+1] - f[n]}{h} + \varepsilon$$

$$f''(t) \approx \frac{f[n+2] - 2f[n+1] + f[n]}{h^2} + \varepsilon$$

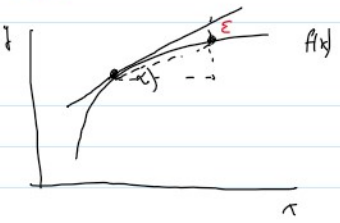
$$f'(t) \approx \frac{f(t+h) - f(t)}{h} \quad \lim_{h \to 0}$$



$$\{y[0], y[1], \dots, y[n], y[n+1], \dots\}$$

$$\Delta_1 \frac{f[n+1] - f[n]}{h} =$$

$$\frac{1}{h}\Delta_1(f[n+1] - f[n]) = \frac{1}{h} \cdot \frac{f[n+2] - f[n+1] - (f[n+1] - f[n])}{h} = \dots$$



Příklad: Spojitý LTI

$$y'(t) = -y(t) \quad i \quad y(0) = 1$$

$$p \cdot Y(p) - y(0) = -Y(p)$$

$$p \cdot Y(p) - 1 = -Y(p)$$

$$p \cdot Y(p) + Y(p) = 1$$

$$Y(p) = \frac{1}{p+1}$$

$$y(t) = e^{-t}$$

Diskrétní systém

Eulerova metoda — dopředná diference

$$y'(t) \approx \frac{y[n+1] - y[n]}{h}$$

$$y'(t) = -y(t) \quad i \quad y(0) = 1$$ — diferenciální rovnice

$$\frac{y[n+1] - y[n]}{h} = -y[n]$$

$$y[n+1] - y[n] = -h\,y[n]$$

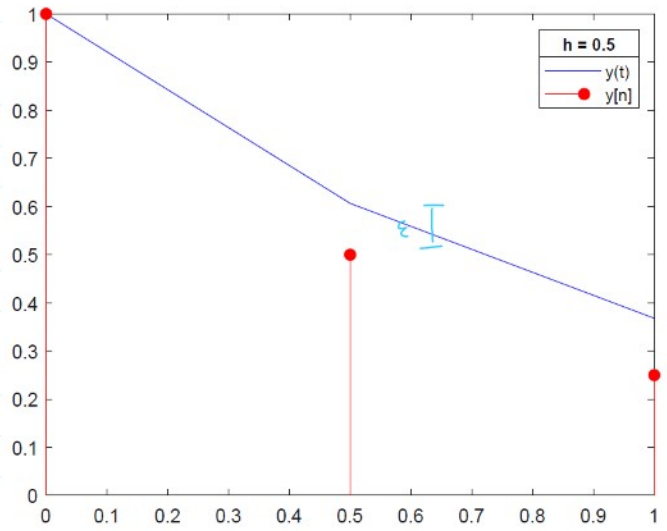
$$y[n+1] = y[n] - h\,y[n]$$ — diferenční rovnice

$h = 0.5$

Matlab

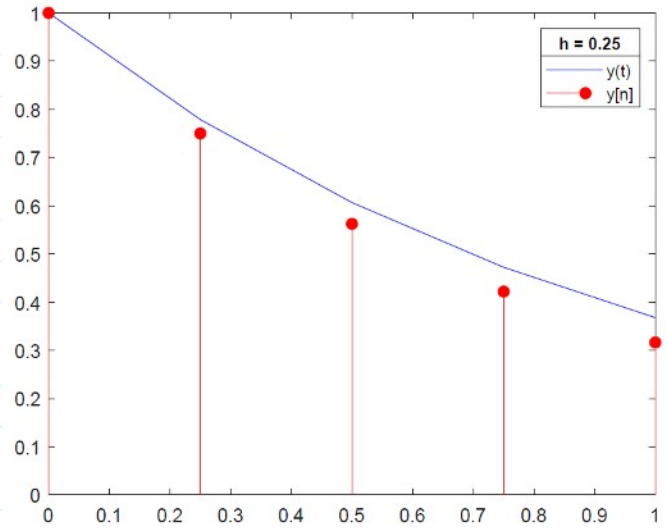
```
clear;
h = 0.5;      % časová diference
t = [0:h:1];
```

$h = 0.5$

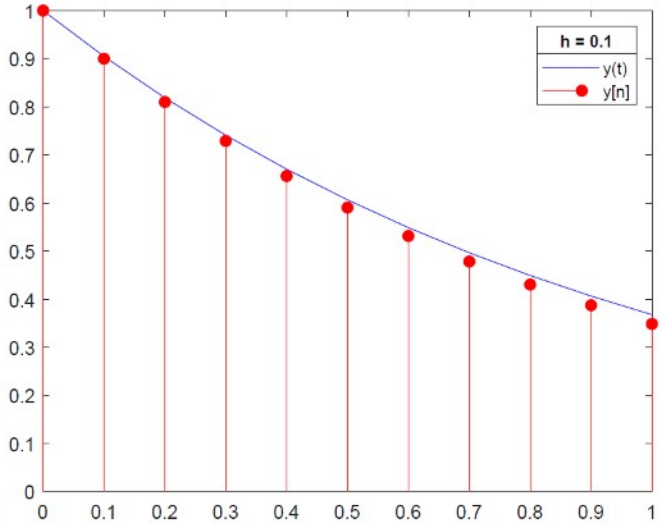



```
clear;
h = 0.5;      % časová diference
t = [0:h:1];
yn(1)=1;
yt(1)=1;
% spojitý systém yt diskretizovaný systém yn
yt = exp(-t);

for i=1:(length(t)-1)
   yn(i+1) = yn(i) - h*yn(i);
end

figure(1);
plot(t,yt,'blue');
hold on
stem(t,yn,'filled','red');
lgd = legend('y(t)','y[n]');
title(lgd,'h = 0.5');
```

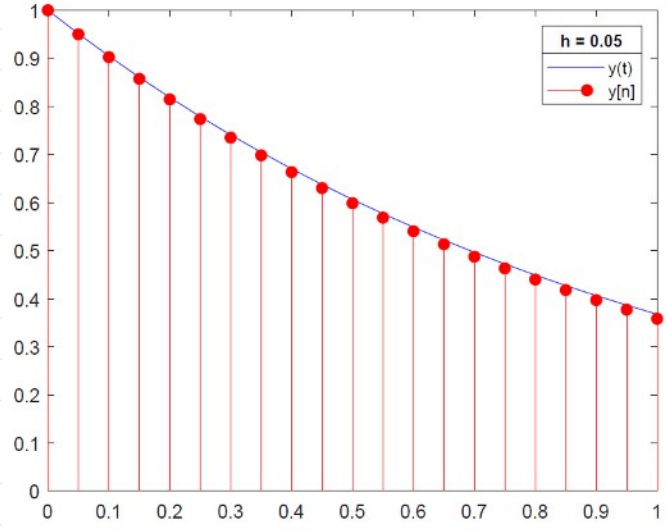
$h = 0.25$



$h = 0.1$



$h = 0.05$



Příklad:

$$y''(t) + 2\cdot y'(t) + y(t) = 1(t) \quad ; \quad y(0)=1 \; ; \quad y'(0)=-1$$

Diferenciální rovnice

$y(t) \rightarrow y[n]$

$1(t) \rightarrow 1[n]$

$$y'(t) \approx \frac{y[n+1]-y[n]}{h}$$

$$y''(t) \approx \frac{y[n+2]-2\cdot y[n+1]+y[n]}{h^2}$$

$$\frac{y[n+2]-2\cdot y[n+1]+y[n]}{h^2} + \frac{2\left(y[n+1]-y[n]\right)}{h} + y[n] = 1[n]$$

$$y[n+2] - 2y[n+1] + y[n] + 2h\,y[n+1] - 2h\,y[n] + h^2 y[n] = h^2 1[n]$$

$$y[n+2] + (2h-2)\,y[n+1] + (h^2-2h+1)\,y[n] = h^2 1[n]$$

$$y[n+2] + 2(h-1)\,y[n+1] + (h-1)^2\,y[n] = h^2 1[n]$$

Diferenční rovnice

$y[n+2]$ ↑ čas $n+2$ — nekauzální systém — $1[n]$ ↑ čas $\rightarrow n$

Eulerova zpětná diference

$$f'(t) \approx \frac{f[n]-f[n-1]}{h}$$

$$f''(t) = \Delta_1\left(\frac{f[n]-f[n-1]}{h}\right) = \frac{f[n]-f[n-1]-\left(f[n-1]-f[n-2]\right)}{h^2} =$$

$$= \frac{f[n]-2\cdot f[n-1]+f[n-2]}{h^2}$$